

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY- GURAJADA VIZIANAGARAM
II B. Tech I Semester Supplementary Examinations, November – 2024
MATHEMATICS-III

(common to all Branches except EEE)

Time: 3 hours

Max. Marks: 75

Answer any FIVE Questions
ONE Question from Each unit
All Questions Carry Equal Marks

- 1 a) Prove that $\nabla(r^2) = 2\vec{r}$ [7]
 b) Find the directional derivative of $\phi = x^2yz + 4xz^2$ in the directional of $2\vec{i} - \vec{j} - 2\vec{k}$ at $(1, -2, -1)$ [8]
 (OR)
- 2 a) Prove that $r^n \vec{r}$ is irrotational. [7]
 b) Evaluate $\int_S \vec{F} \cdot \vec{n} \, ds$ where $\vec{F} = z\vec{i} + x\vec{j} - 3y^2z\vec{k}$ and s is the surface $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$. [8]
- 3 a) Find Laplace transform of $f(t)$ where [7]

$$f(t) = \begin{cases} e^t, & 0 < t < 1 \\ 3, & t > 1 \end{cases}$$

 b) Find Laplace transform of $L \left\{ \int_0^t te^{-t} \sin t \, dt \right\}$ [8]
 (OR)
- 4 a) Find inverse Laplace transform of $\frac{s}{s^4 + 4a^4}$ [7]
 b) Find inverse Laplace transform of $\frac{2}{(s^2+1)(s^2+4)}$ using convolution theorem [8]
- 5 a) Find the Fourier series of $f(x) = 2x$ on $[-1, 1]$ [7]
 b) Find the Half range cosine series of $f(x) = \sin x$ on $[0, \pi]$ [8]
 (OR)
- 6 a) Find the Fourier transform of $f(x)$ defined by $f(x) = \begin{cases} 1 & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$ [7]
 b) Find the Finite Fourier sine transform of $f(x) = \frac{x}{\pi}$ in $(0, \pi)$ [8]
- 7 a) Find partial differential equation by eliminating arbitrary functions from $z = f(2x + 3y) + yg(2x + 3y)$ [7]
 b) Solve the PDE $\sqrt{p} + \sqrt{q} = x^2$ [8]
 (OR)
- 8 a) Find partial differential equation by eliminating arbitrary functions from $\phi(x^2 + y^2 + z^2, xyz) = 0$ [7]
 b) Solve the PDE $p - yq = y^2 - x^2$ [8]
- 9 a) Solve the PDE $(D^2 - 7DD^1 + 12D^{12})z = e^{x-y}$ [7]
 b) Solve the PDE $(D^2 + 2DD^1 + D^{12})z = x^2 + xy + y^2$ [8]

(OR)

- 10 a) Solve the PDE $\frac{\partial^2 z}{\partial x^2} = \frac{\partial z}{\partial y} + 2z$ by the method of separation of variables [7]
- b) Solve the one-dimensional Heat equation is given by $\frac{\partial y}{\partial t} = c^2 \frac{\partial^2 y}{\partial x^2}$ [8]
